Noise Lecture 1

EEL6935
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An IEEE Definition of Noise

• The IEEE Standard Dictionary of Electrical and Electronics Terms defines noise (as a general term) as:
  – *unwanted disturbances superposed upon a useful signal that tend to obscure its information content.*

• Applies both to intrinsic and extrinsic noise
  – *intrinsic noise* is the noise generated inside components themselves
  – *extrinsic noise* designates noise originating elsewhere
Extrinsic Noise
(interference)

• Due to:
  – Cross-talk between circuits
  – E/M (Lightning)
  – Grounding issues
  – Unwanted coupling of AC power supply + harmonics

• Examples (symptoms of)
  – AM/FM radio (especially AM!) during lightning storm
  – “Hum” or “buzz” of electric guitar (from lights, etc)
  – When using portable phone and hear unwanted third party conversation on the line
Intrinsic Noise

• Due to:
  – Resistor thermal noise
  – BJT/diode shot noise
  – MOS thermal (broadband) and flicker (low-freq) noise

• Examples (or rather symptoms of)
  – The “shhhhh” sound of analog TV when signal lost
  – “snow” on the TV screen when signal lost
  – Tape “hiss” heard in old recordings
To Reiterate:
Two General Categories of Noise

• Extrinsic
  – “Interference”, a better term
  – Unwanted signals coupled from sources outside, not due to circuit elements themselves
  Solutions: Topology choice, grounding, shielding

• Intrinsic
  – “Fundamental noise”
  – Inherent to all active devices, resistors
  – Statistical in nature
  Solution: Careful design to minimize effect
This lecture focuses on intrinsic sources of noise

(but both have major impact in bio applications)

From here on “noise” implies intrinsic noise, unless specified.
Noise

• Noise is random in amplitude and phase
• Possible to predict the “randomness” of noise
  – Mean (often is zero)
  – Standard deviation
    • Equal to the RMS value of the noise

Noise waveform and Gaussian distribution of noise amplitudes
Gaussian Noise Distribution

\[ \rho = \frac{e^{-\left( \frac{V^2}{2 \cdot (V_{RMS})^2} \right)}}{V_{RMS} \sqrt{2\pi}} \]

Probability Density (\( \rho \))

Instantaneous Noise Amplitude
Gaussian Noise Distribution

Recall that $V_{RMS}$ is the same as the standard deviation, $\sigma$

Thus, the instantaneous noise amplitude is within +/- 3$\sigma$ ($3 \times V_{RMS}$) ~99.7% of the time.
Noise

- Examples mentioned show that problems due to noise are apparent at the output.

- Sources of noise unique to low-level circuitry, typically input stage.

\[
V_{OUT} \approx V_{NOISE1} \cdot A_1 \cdot A_2
\]

Assuming \(A_1\) is much greater than 1, \(V_{NOISE1}\) dominates and we can ignore the output noise contribution of \(V_{NOISE2}\).
Thermal Noise
(aka Johnson noise, Nyquist noise)

- Due to random motion of electrons in conductor when above absolute zero temperature

From Nyquist:

\[ P_{AVAIL} = k \cdot T \cdot \Delta f \]

\( k = 1.38 \times 10^{-23} \) J/K (Boltzmann’s constant)

\( T \) is temp in K

\( \Delta f \) is the **noise bandwidth** ≠ 3dB bandwidth

\[ V_{NOISE} = ? \]
Thermal Noise

Find $V_{\text{noise}}$ from Available Noise Power

**Eqn 1.**

$$P_{\text{AVAIL}} = k \cdot T \cdot \Delta f$$

Resistor Noise Model

Conjugate Match for max power transfer; $P_{\text{OUT}}$ under this condition is $P_{\text{AVAIL}}$. Find expression for $P_{\text{AVAIL}}$ in terms of $V_{\text{noise}}$

$$V_{\text{OUT}} = V_{\text{noise}} \cdot \frac{R_L}{R_L + R} = \frac{V_{\text{noise}}}{2}$$

$$I_{\text{OUT}} = \frac{V_{\text{noise}}}{R_L + R} = \frac{V_{\text{noise}}}{2 \cdot R}$$

$$P_{\text{AVAIL}} = P_{\text{OUT}} = V_{\text{OUT}} \cdot I_{\text{OUT}} = \frac{V_{\text{noise}}^2}{4 \cdot R}$$

**Eqn 2.**

$$P_{\text{AVAIL}} = \frac{V_{\text{noise}}^2}{4 \cdot R}$$
Thermal Noise

Find $V_{\text{NOISE}}$ from Available Noise Power

**Eqn 1.**

$$P_{\text{AVAIL}} = k \cdot T \cdot \Delta f$$

**Eqn 2.**

$$P_{\text{AVAIL}} = \frac{V_{\text{NOISE}}^2}{4 \cdot R}$$

$V_{\text{NOISE}} = ?$

Let Eqn 1. = Eqn 2. Find $V_{\text{NOISE}}$

$$k \cdot T \cdot \Delta f = \frac{V_{\text{NOISE}}^2}{4 \cdot R}$$

$$V_{\text{NOISE}} = \sqrt{4kTR\Delta f}$$

Note: Thermal noise applies only to true physical resistances, anything that represents energy loss from a system has thermal noise.

$r_\pi$ from BJT model does NOT contribute thermal noise.
Thermal Noise

\[ V_{\text{NOISE}} = \sqrt{4kTR\Delta f} \]

\[ I_{\text{NOISE}} = \sqrt{\frac{4kT\Delta f}{R}} \]

Resistor Noise Models

(From Norton’s theorem)
Thermal Noise

What about $R$ in series?

$$\sqrt{V_{N1}^2 + V_{N2}^2} = \sqrt{4kT\Delta f R_1 + 4kT\Delta f R_2}$$

$$\sqrt{4kT\Delta f (R_1 + R_2)}$$
Thermal Noise

What about R in parallel?

\[ \sqrt{I_{N1}^2 + I_{N2}^2} = \sqrt{\frac{4kT\Delta f}{R_1} + \frac{4kT\Delta f}{R_2}} = \sqrt{\frac{4kT\Delta f}{R_1 \cdot R_2}} + \frac{4kT\Delta f \cdot R_2}{R_1 \cdot R_2} \]

\[ = \sqrt{\frac{4kT\Delta f}{R_1 // R_2}} \]
3dB Bandwidth

- Typically, the bandwidth of a filter is specified in terms of 3db (half-power) bandwidth
  - For a given transfer function, bandwidth spans the frequency range where the magnitude is greater than 3dB down from maximum gain.
- Can be easily measured by driving the circuit with a sinusoidal source and monitoring output level
Noise Bandwidth

• NOT same as 3dB (half-power) bandwidth

• Noise bandwidth – defined in terms of the voltage-gain squared (power gain)
  – Defined for a system with uniform gain throughout passband and zero gain outside
    • Shaped like ideal “brick-wall” filter
  – Since real systems exhibit practical roll-offs, we need to define bandwidth in a manner consistent with the noise equations
Noise Bandwidth

\[ \Delta f = B = \frac{1}{|A_0|^2} \int_0^\infty |A(f)|^2 \, df \]

Actual Response and equivalent noise bandwidth (low-pass). Drawn in linear scale.
Noise Bandwidth Example

\[ f_0 = \frac{1}{2\pi \cdot RC} \]

\[ |A(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}} \]

\[ \Delta f = \frac{1}{|A_0|^2} \int_0^\infty |A(f)|^2 df \]

\[ \Delta f = \int_0^\infty \sqrt{1 + \left(\frac{f}{f_0}\right)^2} df = \int_0^\infty \frac{f}{f_0^2 + f^2} df \]

Using trigonometric substitution: let \( f = f_0 \cdot \tan \theta \) so \( df = f_0 \cdot \sec^2 \theta \cdot d\theta \)

\[ \Delta f = f_0 \int_0^{\pi/2} d\theta \]

\[ \Delta f = \frac{\pi}{2} f_0 \]

Noise bandwidth is 1.57*BW_{3dB} for circuits with 1\textsuperscript{st} order roll-off
Relating Noise & 3dB Bandwidths

- Using circuit roll-off behavior (based on number of poles) can convert 3dB bandwidth to noise bandwidth
  - Use conversion factor

<table>
<thead>
<tr>
<th># of Poles</th>
<th>Δf/BW_{3db}</th>
<th>High Frequency roll-off (dB/decade)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.57</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>1.22</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>1.15</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>1.13</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>1.11</td>
<td>100</td>
</tr>
</tbody>
</table>
Thermal Noise vs. R and $\Delta f$

![Graph showing thermal noise voltage versus resistance and bandwidth](image)
Noise Spectral Density

- Noise Spectral Density is the mean-square value of the noise per unit bandwidth
  - Can be defined in terms of $V$ or $I$
  - Noise spectral density of thermal noise is independent of frequency

\[ S_v(f) = \frac{V_{\text{noise}}^2}{\Delta f} = 4kTR \]
\[ S_i(f) = \frac{i_{\text{noise}}^2}{\Delta f} = \frac{4kT}{R} \]

Thermal noise is described as “white noise” because the energy is equal across all frequencies, an analogy to white light (equal light energy over all wavelengths).
“Spot noise”

- Spot noise – RMS value of the noise in a noise bandwidth of 1Hz.
- Units of Volts/sqrt(Hz) or Amps/sqrt(Hz)

\[
\frac{v_{\text{noise}}}{\sqrt{\Delta f}} = \sqrt{S_v(f)} = \sqrt{4kTR}
\]

\[
\frac{i_{\text{noise}}}{\sqrt{\Delta f}} = \sqrt{S_i(f)} = \sqrt{\frac{4kT}{R}}
\]
Example: Thermal Noise

- What is “spot noise” of 1kΩ resistor at 300K?
- Use this to find $V_{\text{noise}}$ for 100k Ω in 1MHz bandwidth
- $4kT$ is $1.6 \times 10^{-20}$ at room temp (290K)

$$
\frac{V_{\text{noise}}}{\sqrt{\Delta f}} = \sqrt{4kT \cdot R} = \sqrt{1.6 \cdot 10^{-20} \cdot 1000}
$$

= $4.07$ nV/sqrt(Hz)
Example: Thermal Noise Cont.

• What is “spot noise” of 1kΩ resistor at 300K?
• Use this to find $V_{\text{noise}}$ for 100kΩ in 1MHz bandwidth
• $4kT$ is $1.6 \times 10^{-20}$ at room temp (290K)

For the 100kΩ resistor, $V_{\text{noise}}$ is:

$$v_{\text{noise}} = \frac{V_{\text{noise}}}{\sqrt{\Delta f}} \cdot \sqrt{\Delta f} \cdot \sqrt{R} = 4 \frac{nV}{\sqrt{Hz}} \cdot \sqrt{10^6} \cdot \sqrt{100 \cdot 10^3}$$

$$V_{\text{noise}} = 1.27 \text{ mV}_{\text{rms}}$$
Noise Floor

Going back to Nyquist’s expression for $P_{\text{AVAIL}}$ at room temperature (290K):

$$P_{\text{AVAIL}} = k \cdot T \cdot \Delta f = 4 \cdot 10^{-21} \text{ (Watts)}$$

Put in terms of dBm:

$$10 \cdot \log\left(\frac{4 \cdot 10^{-21}}{10^{-3}}\right) = -174 \text{ dBm}$$

Minimum noise level that is practically achievable in a system operating at room temperature.

Thermal noise represents a minimum level of noise.
Ideal Amp with Gain = A

\( R_{\text{out}} \neq 0 \)

\[ \sqrt{4kT R_1 \Delta f} \]

\[ \sqrt{4kT R_{\text{load}} \Delta f} \]
$$R_{\text{out}}: \text{non-zero}$$

(Semi) Ideal Amplifier

$$\sqrt{4kT R_{\text{load}} \Delta f}$$

$$\sqrt{4kT R_1 \Delta f}$$
Shot Noise

- Present in diodes, transistors
  - first observed in vacuum tubes
- Current flow across a potential barrier
  - DC Current is actually the sum of many discrete events when a carrier crosses barrier

\[ I_{SHOT} = \sqrt{2qI_{DC}\Delta f} \]

- RMS noise current, also “white”
- \( q \) is the electronic charge 1.6 x 10\(^{-19} \) Coulombs
- \( I_{DC} \) is bias current in Amps
- \( \Delta f \) is the noise bandwidth
Avalanche Noise

• Due to Zener or avalanche breakdown in a PN junction
• When breakdown occurs EHPs created
• Results in noise produced that is much greater than that of shot noise of same current
• Be cautious with zener based voltage references if noise is a concern
1/f Noise

• Low-frequency noise, NOT “white”
• AKA flicker noise
• Associated with contamination and crystal defects in all active devices
  – Also present in carbon resistors (consider metal film instead)
1/f Noise (see Gray/Meyer)

- Note inverse dependence on frequency

\[ i_d^2 = K \cdot \frac{I_D^a}{f} \cdot \Delta f \]

- \( I_D \) is the drain bias current (this is for long channel MOSFETs)
- \( K \) is a constant based on the device/technology
- \( a \) is a constant between 0.5 and 2
Op Amp Noise Relationships: 1/f Noise, RMS Noise, and Equivalent Noise Bandwidth

"1/f" NOISE

The general characteristic of op amp current or voltage noise is shown in Figure 1 below.

**Figure 1: Frequency Characteristic of Op Amp Noise**
Example
E.I.N. Model

Figure 9-4. A noisy network modeled by the addition of an input noise voltage and current source.
Figure 9-6. Typical total equivalent noise voltage curves for three types of devices.
Figure 9-7. Total equivalent input noise voltage $V_n$, for a typical device. The total noise voltage is made up of three components (thermal noise, $V_\pi$, and $I_eR_s$) as was shown in Eq. 9-30.
“kT/C Noise”
Resources

• EEE5320 & EEE6321 Notes (Dr. Fox)
• Low-Noise Electronic System Design by Motchenbacher and Connelly
• Electronic Noise and Interfering Signals by Vasilesescu
• Noise Reduction Techniques in Electronic Systems by Ott